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Patent Valuation under Fragile Institutional Enforcement: A Continuous-Time Markov Approach

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Abstract

We build a tractable model that links institutional dynamics with the private value of innovation. Our approach differs from much of the existing literature in that an inventor does not retain a perpetual monopoly over its use, and the cash flows generated from a new idea are uncertain. In our framework the relevant dimension of institutional quality is enforcement strength. We model institutional strength as a two-state continuous-time Markov chain. This makes the cash flows from innovation stochastic and state-dependent, and hence the incentive to innovate varies with the strength of enforcement regime. Countries alternate between periods of strong and weak enforcement, reflecting irregular political and legal events such as reforms, leadership changes, or crises. Our model shows how institutional fragility can alter the incentive to innovate and connects institutional dynamics with cross-country differences in standard of living.

JEL: O31; O33; O34; O43;

Keywords: institutions, innovation, patents, continuous-time Markov chain, economic growth

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1. Introduction

The neoclassical growth model highlights technological progress as the engine of economic growth (Solow, 1956). Ideas, the most important ingredient of technologies, can flow rapidly across countries and machines that embody better technologies can be imported by poor countries (Acemoglu and Zilibotti, 2001). If so, why do farms in advanced economies deploy high-tech machinery and modern inputs, while agriculture in developing and low-income countries remains predominantly labor-intensive even today? Why is it that some countries are so much more innovative than others? What explains growth miracles and growth disasters? We all know that history has proceeded very differently for peoples from different parts of the globe. Some parts of the world developed industrial societies with metal tools, other parts developed only farming societies, and still others retained societies of hunter-gatherers with stone tools (Diamond, 1997). [1, 2]

To answer these questions we need to first understand where the technological progress that underlies this growth comes from. The endogenous growth literature focused specifically on this very issue by trying to understand the economic forces underlying technological progress (Romer, 1990), (Grossman and Helpman, 1991), and (Aghion and Howitt, 1992). An important contribution of this work is the recognition that technological progress occurs as profit-maximizing firms or inventors seek out new ideas in an ef-

[1] A seemingly compelling argument involves implicitly or explicitly differences in innate ability among peoples. However, sound evidence for the existence of human differences in intelligence that parallel human differences in technology is lacking. A large body of evidence shows that the same workers earn multiples more when they move to rich countries, pointing instead to differences in organizational practices and institutions as the primary drivers of measured productivity gaps rather than innate skill differences (Clemens et al., 2012). It is interesting that, in a relatively small sample of members of the U.S. National Academy of Sciences and National Academy of Engineering, foreign-born scientists have tended to earn significantly more on average than native ones (Guellec and Cervantes, 2002). To the extent that the best talent leaves, there may be nontrivial implications for the developing country's ability to implement technological progress and move activities up the value chain.

[2] (Acemoglu and Zilibotti, 2001) focus on the skill mix of the population. In their model richer countries are inventing technologies that are optimal for a highly skilled workforce. Poorer countries can imitate these technologies or acquire them, but they cannot exploit its full potential. However, if poor countries enforce property rights, then firms in richer countries would find it profitable to develop technologies appropriate for poorer countries.

fort to capture some of the social gain these new ideas generate in the form of profit. Adam Smith wrote that "it is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their own interest" (Smith, 1776).[3]

Legendary investor Warren Buffett stated once that, "society is responsible for a very significant percentage of what I've earned." Context matters, he argued. "If you stick me down in the middle of Bangladesh or Peru or someplace, you find out how much this talent is going to produce in the wrong kind of soil," Buffet added. "I work in a market system that happens to reward what I do very well - disproportionately well" (Buffett and Lowe, 1997). The quote underscores an obvious fact: that not all countries afford people the same property rights, opportunities or guarantees. Luck matters! Undoubtedly, social infrastructure (protection of private property rights, rule of law, enforcement of contracts, lack of corruption, taxation, openness to trade and to flow of capital, etc) is part of the answer to the different wealths of nations (North, 1991), (Hall and Jones, 1999), and (Acemoglu et al., 2005).[4]

Moreover, when intellectual property rights are credible and enforceable across jurisdictions, new technologies can diffuse rapidly; even with frictions, diffusion limits how far countries can fall behind the frontier countries (Acemoglu et al., 2005). The experiences of Japan and South Korea, both of which transitioned from relatively poor to relatively rich within a few decades, underscore this point. Individual ability did not change overnight; rather, policy and institutional reforms encouraged domestic investment in physical capital, attracted foreign direct investment and technology transfer, and fostered human-capital accumulation. The transistor invented at Bell Labs in 1947, leapt 8000 miles to launch an electronics industry in Japan - but it did not

[3]It must be said that this is not true for all inventions. For example, in 1942, in the middle of World War II, the United States government set up the Manhattan Project with the explicit goal of inventing the technology required to build an atomic bomb before Nazi Germany. Furthermore, many technologies were developed by people driven by curiosity, in the absence of any initial demand for the product they had in mind (Diamond, 1997). Still other devices, invented to serve one purpose, eventually found most of their use for other - accidental by-products.

[4]The clearest evidence for this view comes from pairs of countries that divide essentially the same environment but have very different institutions and, associated with those institutions, different standards of living. Examples include, comparison of South Korea with North Korea, the former West Germany with the former East Germany, and Israel with its Arab neighbors.

make the shorter leap to found new industries in Senegal.[5, 6]

Against this backdrop, we build a tractable model that links institutional dynamics with the private value of innovation. Our approach differs from much of the existing literature ((Romer, 1990), (Grossman and Helpman, 1991), (Aghion and Howitt, 1992)) in that the inventor of a product or technique does not retain a perpetual monopoly over its use, and the cash flows generated from a new idea are uncertain. This is more realistic as an inventor's position would erode over time as competitors learned about the new product or technique and imitated it or created close substitutes (Arrow, 1962) and (Dixit and Stiglitz, 1977). Monopoly power might also diminish over time because patent protection was only temporary ((Nordhaus, 1969) and (Barro and Sala-i Martin, 2003)).

A closely related channel, and the one central to our analysis is institutional strength. The social infrastructure of an economy - the rules and regulations and the institutions that enforce them - is a primary determinant of the extent to which individuals are willing to make the long-term investment in capital, skills, and technology. Economies in which the government provides an environment that encourages production are extremely dynamic. Those in which the government abuses its authority are correspondingly less successful. The empirical evidence produced by (Hall and Jones, 1999); (Acemoglu et al., 2005); (Easterly and Levine, 2002); among others, supports these claims.

In what follows we abstract from these other forces that dissipate rents

[5]Indeed, surveys of foreign companies doing business in emerging markets are revealing. They regularly show that the issue of property rights being weak is often the number one concern they have in operating in these markets. And this becomes increasingly relevant as the assets that differentiate firms are intangible - technologies, brands, trade secrets, etc. Weaker patent protection implies lower expected profit for these firms. Corruption, bribery, and expropriation risk erode those expectations, depressing investment and living standards. See (Park, 2008) for example, who constructed an index of the strength of patent protection for 110 countries over the period 1960–2005, by coding national patent laws according to the extent of coverage of different technologies, membership in international treaties, potential to lose protection, presence of enforcement mechanisms and duration.

[6]There is increasing recognition that good institutions are not a random variable that could have popped up anywhere around the globe with equal probability (Bockstette et al., 2002). Moreover, this literature emphasizes the role of other proximate variables besides good institutions, such as, histories of state societies and role of agriculture, in order to explain the cross-country variations in the standard of living we observe (Diamond, 1997).

and focus instead on variations in enforcement strength. This keeps the model tractable while isolating the key question: how changes in enforcement strength affect the share of monopoly rents that an innovator can capitalise. In our framework the relevant dimension of institutional quality is enforcement strength. We model enforcement strength as a two-state continuous-time Markov chain (CTMC).[7] This makes the cash flows from innovation stochastic and state-dependent, and hence the incentive to innovate varies with the enforcement regime. Countries alternate between periods of strong and weak enforcement, reflecting irregular political and legal events such as reforms, leadership changes, or crises. A continuous-time Markov chain provides a natural way to capture this discreteness and randomness while linking the frequency of transitions to the private value of innovation.

Our model yields three main results. First, it nests the benchmark valuation formula when enforcement regime is strong and is known with certainty. Second, under stochastic enforcement we obtain closed-form expressions for both the conditional price (given the regime at the valuation date) and the unconditional price (averaging across regimes). Third, we introduce the concept of the valuation rent, the additional capitalised value that arises from the possibility of strong enforcement relative to a permanently weak enforcement, and show that it equals the flow monopoly rent scaled by a rent multiplier. The multiplier converges in the long run to the fundamental value of an asset weighted by the stationary share of time under strong enforcement, while in the short run it deviates according to the initial regime. This decomposition highlights how enforcement dynamics shape the valuation of a patent in an empirically tractable way.

The rest of the paper is organized as follows. Section 2 the valuation of an asset (patents) in a deterministic setting (Romer, 1990), (Grossman and Helpman, 1991), (Aghion and Howitt, 1992). Section 3 extends this framework to a stochastic case. We model institutional strength as a two-state continuous-time Markov chain that makes the cash flows from innovation stochastic and state-dependent, and hence the incentive to innovate varies with the enforcement regime. The framework shows how institutional fragility can alter the incentive to innovate. Section 4 concludes.

[7]Empirically, Markov-chain models of political or institutional regimes have been used to study the resilience of democracies, persistence of collapses, and transitions between institutional states. The data support modeling countries' institutional positions as states with transition probabilities across time (Imam and Temple, 2023).

2. Fundamental Valuation of a Patent - The Deterministic Case

In order to motivate research, successful innovators have to be compensated in some manner. The basic problem is that the creation of a new design or blueprint is costly (R&D expenditure) but could then be used in a non-rival way by all potential users of the design. It would be efficient, ex post, to make the design freely available to all, but this practice fails to provide the ex ante incentives for further discoveries.

Romer (1990) in his seminal paper considers an institutional setup in which the inventor of a new design retains a perpetual monopoly right (enforced through explicit patent protection) over the production and sale of a capital goods that uses his or her design. The flow of monopoly profits provides the incentive for invention.

Therefore, the price of a patent (P_A) is the present value of profit flow i.e.,

$$P_A(t) = \int_t^{\infty} \pi(s) e^{-\int_t^s r(\tau)d\tau} ds \quad , \tag{2.1}$$

where $\pi(s)$ is the profit flow of date s and r is the risk-free interest rate. Exploiting the Leibniz's rule for differentiation of a definite integral, we get[8]

$$r(t)P_A(t) = \pi(t) + \dot{P}_A(t) \quad . \tag{2.2}$$

The left-hand side of equation (2.2) is the interest earned from investing, P_A, in a bank; the right-hand side is the profits plus the capital gain or loss that results from the change in the price of the patent. Equation (2.2) is the no arbitrage condition. In fact, the absence of arbitrage opportunities is a fundamental principle underlying the modern theory of financial asset

[8] The Leibniz formula if the limits of integration are function of x and $f(x, y)$ is a function of two variables that can be integrated with respect to 't' and differentiated with respect to x:

$$F'(x) = \int_{a(x)}^{b(x)} f_x(x,t)dt + f(x, b(x))b'(x) - f(x, a(x))a'(x) \quad .$$

if $a(x) = a$ is a constant and $b(x) = x$, the rule simplifies to:

$$F'(x) = f(x,x) + \int_a^x f_x(x,t)dt \quad .$$

pricing.

The reasoning is as follows. Suppose an investor has 'x' dollars to invest for 1-year. He has two options. First, he can put the money in a bank for 1-year and earn the interest rate, r. Alternatively, he can purchase a patent for 1-year, earn the profits in that period, and then sell the patent. In equilibrium, it must be the case that the rate of return from both of these investments is the same. If not, everyone would jump at the more profitable investment, driving its return down.

Rewriting equation (2.2) yields,

$$r(t) = \frac{\pi(t)}{P_A(t)} + \frac{\dot{P}_A(t)}{P_A(t)} \quad . \tag{2.3}$$

In steady-state, r is a constant in Romer's model. Therefore, $\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{P}_A(t)}{P_A(t)}$. Furthermore, the rate of growth of profit equals the growth rate of population ($n > 0$).[9]

Finally, substituting $\dot{P}_A(t)/P_A(t) = n$ into equation (2.3) above and rearranging yields the price of a patent along the balanced growth path (BGP), i.e.,

$$P_A(t) = \frac{\pi(t)}{r - n} \quad . \tag{2.4}$$

This equation is the present discounted value of cash flows in a deterministic setting when the strength of the enforcement regime is known with certainty. It tells us what influences the price of a patent in equilibrium. A higher r indicates that the present discounted value of profits is lower which naturally leads to a lower price for the patent.[10] In contrast, if n is higher this would encourage more R&D activity, as the scale of the economy (market size) is growing quickly, increasing demand for new products and potential profits. This in turn leads to a higher price for the patent.

[9] In Romer (1990), population growth is assumed to be zero i.e., $n = 0$. Therefore, the price of a patent along the balanced growth path is given by, $P_A(t) = \left(\frac{1}{r}\right)\pi(t)$.

[10] An alternative way to understand this is to think of r as representing the return a potential investor could earn by putting their money in a bank for 1-year, rather than purchasing a patent.

3. Valuation of a Patent - The Stochastic Case

The benchmark model in Section 2 values a patent as if monopoly profits were permanent and the enforcement regime is known with certainity. In practice, those rents are fragile (Arrow, 1962) and (Nordhaus, 1969); (Barro and Sala-i Martin, 2003) stressed how imitation and competition erode private returns; (Dixit and Stiglitz, 1977) showed how entry in differentiated-product markets dilutes market power. A closely related channel, and the one central to our analysis, is enforcement strength. When courts, regulators, or contract mechanisms are weak, rivals can copy, pirate, or breach agreements at a low cost, and the innovator's private return shrinks. We abstract from the many forces that dissipate rents and focus instead on variation in enforcement strength, modelling it as a stochastic process that alternates between two regimes. This keeps the model tractable while isolating the key question: how changes in enforcement strength affect the share of monopoly rents that an innovator can capitalise.

3.1. Modified arbitrage under stochastic enforcement

We model the enforcement regime $X(t)$ at date t, with $X(t) = 1$ denoting strong enforcement and $X(t) = 0$ denoting weak enforcement.[11]

Profits in regime i take the form

$$\pi_i(t) = \pi_i e^{nt}, \qquad i \in \{0, 1\}, \tag{3.1}$$

where $n > 0$ is the population growth rate and $\pi_1 > \pi_0$. This exponential scaling reflects the standard BGP assumption that market size, and hence the demand for a patented product, expands at the growth rate of population. The profit gap,

$$\Delta\pi(t) := \pi_1(t) - \pi_0(t) = (\pi_1 - \pi_0)e^{nt}, \tag{3.2}$$

is the *monopoly rent*: the incremental flow of profits available only when enforcement is effective. Because the enforcement state X(t) is stochastic, the profit flow at date t is itself random. We denote the realised profit by,

$$\Pi(t) := \pi_{X(t)} e^{nt}. \tag{3.3}$$

[11] Appendix A provides a short introduction to continuous-time Markov chains.

The price of a patent at date t is the expected discounted stream of future cash flows. With full information and a constant riskless rate r,

$$P_A(t) = \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\, \pi_{X(s)} e^{ns}\, ds\right]. \tag{3.4}$$

Changing variables to $u = s - t$ gives,

$$P_A(t) = \mathbb{E}\left[\int_0^{\infty} e^{-ru}\, \pi_{X(t+u)}\, e^{n(t+u)}\, du\right] = e^{nt}\, \mathbb{E}\left[\int_0^{\infty} e^{-(r-n)u}\, \pi_{X(t+u)}\, du\right]. \tag{3.5}$$

The price of a patent in equation 3.5 clearly decomposes into a deterministic growth component and a stochastic component driven by enforcement. Thus the model nests the standard benchmark as a special case.

Enforcement is modelled as a two–state continuous-time Markov process. From the weak regime the system switches to strong enforcement with instantaneous rate, λ; from the strong regime it switches to weak with rate, μ. Agents observe $X(t)$ in real time, the patent price equals the fundamental present value of cash flows, and values adjust continuously at regime switches. Figure 1 illustrates the regime dynamics.

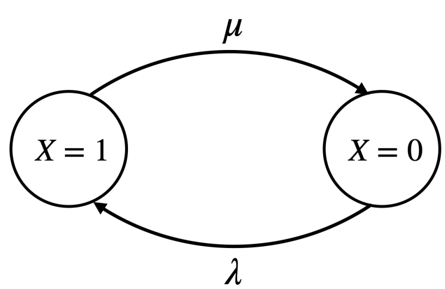

Figure 1: State–transition diagram for institutional enforcement. The economy alternates between two regimes: weak enforcement ($X = 0$) and strong enforcement ($X = 1$). An arrow labeled 'λ' denotes the instantaneous rate of moving from weak to strong enforcement, while 'μ' denotes the rate of moving from strong to weak enforcement. These rates summarize the frequency of institutional reforms, reversals, or other political–legal events that shift the effectiveness of enforcement.

Under these maintained assumptions the standard no-arbitrage condition applies:[12]

$$r\,P_A(t) = \Pi(t) + \dot{P}_A(t), \tag{3.6}$$

which follows by differentiating the integral representation and imposing the transversality condition that selects the fundamental or no bubbles solution.

Two valuation objects are useful. The first is the *conditional price*: the expected value of a patent given the enforcement regime observed at date t. This object is ex-post with respect to the realised regime at the valuation date but forward-looking over future transitions. It shows how the value of innovation depends on the institutional environment in place at the time of valuation. When enforcement is strong, the expected value is higher because profits are more appropriable; when enforcement is weak, the expected value is lower but not zero, since some profits accrue even under weak enforcement and the regime may improve in the future. The second is the *unconditional expected price*, which averages over the stochastic evolution of enforcement strength. This provides a regime-agnostic measure of patent value, suitable when the current state is not directly observed or when one wishes to compare values across countries or periods.

Our contribution is to identify a third object, the *valuation rent*, defined as the additional capitalised value that arises from the possibility of strong enforcement relative to permanently weak enforcement. Denote the weak-enforcement benchmark by

$$P_A^0(t) := \frac{\pi_0(t)}{r-n}, \tag{3.7}$$

and define

$$\Delta P(t) := \mathbb{E}[P_A(t)] - P_A^0(t). \tag{3.8}$$

Because $\Delta\pi(t)$ is the flow monopoly rent, it is natural to measure $\Delta P(t)$ per unit of flow rent. We therefore introduce the *rent multiplier*,

$$M(t) := \frac{\Delta P(t)}{\Delta\pi(t)}, \tag{3.9}$$

which expresses how a unit of flow monopoly rent is capitalised into asset value once enforcement dynamics, growth, and discounting are taken into

[12] We can solve for the value of the patent by integrating this differential equation as illustrated in Appendix B

account.

We next derive closed-form valuation formulas. Theorems 3.1 and 3.2 presents the conditional price, followed by the unconditional price and the rent multiplier.

Theorem 3.1 (Conditional price)**.** *Suppose $r > n$ and $\lambda, \mu > 0$.* [13] *Then*

$$\mathbb{E}[P_A(t) \mid X(t) = 0] = e^{nt} \left[\frac{\bar{\pi}}{r-n} - \frac{\lambda}{\lambda+\mu} \cdot \frac{\Delta\pi}{r-n+\lambda+\mu} \right],$$

$$\mathbb{E}[P_A(t) \mid X(t) = 1] = e^{nt} \left[\frac{\bar{\pi}}{r-n} + \frac{\mu}{\lambda+\mu} \cdot \frac{\Delta\pi}{r-n+\lambda+\mu} \right].$$

Theorem 3.2 (Unconditional valuation and rent multiplier)**.** *Let $p_0(0) = \Pr\{X(0) = 0\}$.*[14] *Then*

$$\mathbb{E}[P_A(t)] = \frac{\bar{\pi}\, e^{nt}}{r-n} - \frac{\Delta\pi\, e^{(n-(\lambda+\mu))t}}{r-n+\lambda+\mu} \left[p_0(0) - \frac{\mu}{\lambda+\mu} \right].$$

Equivalently,

$$M(t) = \frac{\lambda}{\lambda+\mu} \cdot \frac{1}{r-n} - \frac{e^{-(\lambda+\mu)t}}{r-n+\lambda+\mu} \left[p_0(0) - \frac{\mu}{\lambda+\mu} \right].$$

The interpretation of these theorems is straightforward. The rent multiplier has two parts. A permanent component, $(r-n)^{-1}$, which happens to be

[13] Proof sketch of Theorem 3.1. Starting from

$$P_A(t) = e^{nt} \int_0^\infty e^{-(r-n)u}\, \pi_{X(t+u)}\, du,$$

take expectations conditional on $X(t) = j$. The two–state transition probabilities decompose into stationary weights plus an exponential transient that decays at rate $\lambda + \mu$ (see Appendix A). Substituting this representation reduces the valuation to two elementary integrals,

$$\frac{1}{r-n}, \frac{1}{r-n+\lambda+\mu}.$$

Collecting terms yields the conditional formulas in Theorem 3.1. The full proof of this theorem can be found in Appendix C.

[14] The unconditional result in Theorem 3.2 follows by averaging the conditional values using the initial distribution, $p_0(0)$. The proof is illustrated in Appendix D.

the multiplier from the benchmark valuation in section 2 scaled by the stationary share of time spent under strong enforcement; this gives the long-run capitalisation of a unit flow rent. The transient component depends only on the initial regime and decays at rate $\lambda + \mu$. If the economy starts weak the transient lowers current value; if it starts strong it raises it. Stochastic enforcement therefore implies both a permanent scaling of capitalised rents and a short-lived deviation reflecting the starting point. Because $M(t)$ translates flow monopoly rents into asset values, it is the natural comparative-static object for empirical measurement and policy evaluation.

Limiting cases

Several limiting cases clarify the interpretation of the rent multiplier and relate the stochastic model to the standard benchmark.

Permanently strong enforcement. If the process begins in the strong state ($p_0(0) = 0$) and never exits ($\mu = 0$), enforcement is permanently effective. The rent multiplier reduces to

$$M(t) = \frac{1}{r - n},$$

so the model collapses to the standard benchmark with perpetual monopoly protection. United States and the United Kingdom are good examples of this case. While the United States benefited from the Constitution and the Bill of Rights, the United Kingdom developed a separation of powers between the Crown and Parliament, established institutions to secure property rights, and a strong judicial system i.e., social infrastructure that are conducive for production rather than diversion.

Permanently weak enforcement. If the process begins in the weak state ($p_0(0) = 1$) and never upgrades ($\lambda = 0$), enforcement is permanently ineffective. The valuation rent is then identically zero, and the patent price equals the weak–enforcement benchmark. In this environment, innovation incentives vanish. Most of sub-Saharan Africa and South Asia fall under this category. Countries under this category continue to languish at the bottom of the world income distribution.

One-way upgrade. If the process begins weak ($p_0(0) = 1$), upgrades are possible ($\lambda > 0$), but downgrades never occur ($\mu = 0$), then the economy eventually converges to the strong state. The multiplier converges to the standard benchmark value $1/(r - n)$, but the path is tilted downward in the short run. Rapid economic transformation of economies such as Japan and

South Korea since World War II are examples of this case. In the span of a few decades, these countries have moved from a relatively poor, war-weary economy into the very top of the world income distribution.

One-way downgrade. If the process begins strong ($p_0(0) = 0$), downgrades are possible ($\mu > 0$), but upgrades never occur ($\lambda = 0$), then the economy eventually converges to the weak state. The multiplier converges to zero, so initial high values fade over time. This accrues to economies that start of well but fall to a weaker state and get stuck there. Example of this reverse movement is Argentina. For example, at the end of the nineteenth century, Argentina enjoyed a standard of living comparable to Western Europe. But today its per capita income is a fraction of that of Western Europe. Much of this decline is attributed to disastrous policies. Similarly, hyperinflation and general mismanagement of the economy in several Latin American countries during the 1980s or in Zimbabwe more recently fall under this category.

4. Conclusion

This paper develops a framework that links institutional dynamics to the private value of innovation. This keeps the model tractable while isolating the key question: how changes in enforcement strength affect the share of monopoly rents that an innovator can capitalise. In our framework the relevant dimension of institutional quality is enforcement strength. We model institutional strength as a two–state continuous-time Markov chain and obtain closed-form solutions for patent valuation under stochastic enforcement. The framework nests the textbook benchmark (with permanent monopoly rents) as a special case and extends it to environments where enforcement is uncertain.

The central new object is the valuation rent, the additional capitalised value that arises from the possibility of strong enforcement relative to permanently weak enforcement. Because the flow monopoly rent is the profit differential across regimes, the valuation rent equals this flow rent scaled by a rent multiplier. The multiplier has a simple structure: a long-run level determined by the share of time an economy spends under strong enforcement, and a short-run adjustment reflecting initial conditions. The framework highlights how institutional fragility alters the incentive to innovate. In countries where enforcement is strong, patents command higher values and firms invest more in new ideas, and growth is sustained. Where enforcement is weak or erratic, the private return to innovation is lower, the valuation rent shrinks, and the incentive to innovate is weak. Two economies with similar laws on paper can therefore diverge sharply depending on the effectiveness of enforcement or on how they arrive at their current regime.

The framework thus provides a way to connect institutional dynamics with cross-country differences in innovation and living standards. It clarifies which components of enforcement matter to the innovator: the average time spent under effective protection, the profit gap between regimes, and the speed of transition.

Appendix A. Continuous-time Markov chains

This appendix provides a brief introduction to the two–state continuous-time Markov chain (CTMC) used in the main text. A CTMC is a stochastic process $\{X(t) : t \geq 0\}$ taking values in a finite state space S with the property that future evolution depends only on the current state:

$$\Pr\{X(t+u) = i \mid X(t) = j, \mathcal{F}_t\} = \Pr\{X(t+u) = i \mid X(t) = j\}, i, j \in S, \tag{A.1}$$

for all $u \geq 0$, where $\mathcal{F}_t$ is the history up to time t. The dynamics are summarized by a *generator matrix* $Q = (q_{ij})_{i,j \in S}$, whose off-diagonal entries q_{ij} are the transition intensities from state j to state i and whose rows sum to zero.

In the two–state case $S = \{0, 1\}$, write

$$Q = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix}, \tag{A.2}$$

so that the process exits the weak state (0) at rate λ and exits the strong state (1) at rate μ. The transition probabilities

$$p_{ji}(u) = \Pr\{X(t+u) = i \mid X(t) = j\},$$

satisfy the Kolmogorov forward equation $\frac{d}{du}P(u) = QP(u)$ with $P(0) = I$. Solving gives the closed forms,

$$\begin{aligned} p_{00}(u) &= \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)u}, \\ p_{01}(u) &= \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)u}, \\ p_{11}(u) &= \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)u}, \\ p_{10}(u) &= \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)u}. \end{aligned} \tag{A.3}$$

Each transition probability is the sum of a *stationary weight* (the long-run probability of the destination state) and an exponentially decaying transient.

The stationary distribution is,

$$\pi_0^* = \frac{\mu}{\lambda+\mu}, \qquad \pi_1^* = \frac{\lambda}{\lambda+\mu}. \tag{A.4}$$

Intuitively, the process spends fraction π_i^* of time in state i in the long run, while the transient term records how far the system is from that steady mix at time t. The decay rate $\lambda+\mu$ is the mean-reversion speed and equals the sum of the exit hazards.

The probabilities of occupying either state, $\Pr\{X(t)=0\}$ and $\Pr\{X(t)=1\}$, follow from the law of total probability and the two–state transition probabilities. For state 0 we have

$$\begin{aligned} p_0(t) &:= \Pr\{X(t)=0\} \\ &= p_0(0)\, p_{00}(t) + \big(1-p_0(0)\big)\, p_{10}(t) \\ &= \frac{\mu}{\lambda+\mu} + \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big) e^{-(\lambda+\mu)t}. \end{aligned} \tag{A.5}$$

For state 1,

$$\begin{aligned} p_1(t) &:= \Pr\{X(t)=1\} = 1 - p_0(t) \\ &= \frac{\lambda}{\lambda+\mu} - \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big) e^{-(\lambda+\mu)t}. \end{aligned} \tag{A.6}$$

Appendix B. Solving the differential equation

Starting from the arbitrage identity

$$rP_A(t) = \pi(X(t),t) + \dot{P}_A(t), \tag{B.1}$$

we rearrange:

$$\dot{P}_A(t) - rP_A(t) = -\pi(X(t),t). \tag{B.2}$$

Multiplying by e^{-rt} and integrating from t to ∞:

$$\frac{d}{dt}\big(e^{-rt}P_A(t)\big) = -e^{-rt}\pi(X(t),t). \tag{B.3}$$

Integrating both sides,

$$\int_t^\infty \frac{d}{ds}\big(e^{-rs}V(s)\big)\,ds = -\int_t^\infty e^{-rs}\pi(X(s),s)\,ds. \tag{B.4}$$

Using the transversality condition $\lim_{s\to\infty} e^{-rs}V(s) = 0$ gives

$$-e^{-rt}P_A(t) = -\int_t^\infty e^{-rs}\pi(X(s),s)\,ds, \tag{B.5}$$

and hence

$$P_A(t) = \int_t^\infty e^{-r(s-t)}\pi(X(s),s)\,ds, \tag{B.6}$$

which is the integral representation used in the main text.

Appendix C. Proof of Theorem 3.1

By definition of conditional expectation,

$$\mathbb{E}[P_A(t) \mid X(t) = j] = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}\,\pi_{X(s)}(s)\,ds \,\middle|\, X(t) = j\right]. \tag{C.1}$$

Use the partition of states,

$$1_{\{X(s)=0\}} + 1_{\{X(s)=1\}} = 1, \tag{C.2}$$

to write the integrand as a sum over $i \in \{0, 1\}$:

$$\pi_{X(s)}(s) = \sum_{i=0}^{1} \pi_i(s)\,1_{\{X(s)=i\}}. \tag{C.3}$$

Because the integrand is nonnegative, Tonelli's theorem justifies interchanging expectation and integration:

$$\mathbb{E}[P_A(t) \mid X(t) = j] = \sum_{i=0}^{1}\int_t^\infty e^{-r(s-t)}\,\pi_i(s)\,\Pr\{X(s) = i \mid X(t) = j\}\,ds. \tag{C.4}$$

Writing $u = s - t$ and using the time-homogeneity of the CTMC,

$$\mathbb{E}[P_A(t) \mid X(t) = j] = \sum_{i=0}^{1} \int_0^\infty e^{-ru}\, \pi_i(t+u)\, p_{ji}(u)\, du, \tag{C.5}$$

where $p_{ji}(u) = \Pr\{X(t+u) = i \mid X(t) = j\}$ is the u–step transition probability. Substituting $\pi_i(t+u) = \pi_i e^{n(t+u)}$ yields

$$\mathbb{E}[P_A(t) \mid X(t) = j] = e^{nt} \sum_{i=0}^{1} \pi_i \int_0^\infty e^{-(r-n)u}\, p_{ji}(u)\, du. \tag{C.6}$$

Since we have a two–state CTMC with generator

$$Q = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix}, \tag{C.7}$$

the discussion in Appendix A applies and the transition probabilities are given by Equations A.3. Evaluating the integrals proceeds as follows. For $X(t) = 0$,

$$\mathbb{E}[P_A(t) \mid X(t) = 0] =$$
$$e^{nt}\left[\pi_0 \int_0^\infty e^{-(r-n)u} p_{00}(u)\, du + \pi_1 \int_0^\infty e^{-(r-n)u} p_{01}(u)\, du\right]. \tag{C.8}$$

Substituting the transition probabilities,

$$= e^{nt}\left[\pi_0 \int_0^\infty e^{-(r-n)u}\left(\tfrac{\mu}{\lambda+\mu} + \tfrac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)u}\right) du\right.$$
$$\left. + \pi_1 \int_0^\infty e^{-(r-n)u}\left(\tfrac{\lambda}{\lambda+\mu} - \tfrac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)u}\right) du\right]. \tag{C.9}$$

Split each integral into its constant part and its exponential part:

$$= e^{nt}\left[\pi_0\left(\tfrac{\mu}{\lambda+\mu} \int_0^\infty e^{-(r-n)u} du + \tfrac{\lambda}{\lambda+\mu} \int_0^\infty e^{-(r-n+\lambda+\mu)u} du\right)\right.$$

$$+\pi_1\Big(\tfrac{\lambda}{\lambda+\mu}\int_0^\infty e^{-(r-n)u}du - \tfrac{\lambda}{\lambda+\mu}\int_0^\infty e^{-(r-n+\lambda+\mu)u}du\Big)\Bigg]. \tag{C.10}$$

The integrals evaluate to $(r-n)^{-1}$ and $(r-n+\lambda+\mu)^{-1}$, so

$$= e^{nt}\Bigg[\frac{\mu}{\lambda+\mu}\cdot\frac{\pi_0}{r-n} + \frac{\lambda}{\lambda+\mu}\cdot\frac{\pi_0}{r-n+\lambda+\mu}$$

$$+\frac{\lambda}{\lambda+\mu}\cdot\frac{\pi_1}{r-n} - \frac{\lambda}{\lambda+\mu}\cdot\frac{\pi_1}{r-n+\lambda+\mu}\Bigg]. \tag{C.11}$$

Collecting terms gives

$$\mathbb{E}[P_A(t)\mid X(t)=0] = e^{nt}\left[\frac{\bar\pi}{r-n} - \frac{\lambda}{\lambda+\mu}\cdot\frac{\Delta\pi}{r-n+\lambda+\mu}\right], \tag{C.12}$$

where $\bar\pi = \frac{\mu}{\lambda+\mu}\pi_0 + \frac{\lambda}{\lambda+\mu}\pi_1$ and $\Delta\pi = \pi_1 - \pi_0$.

The case $X(t)=1$ is analogous, substituting $p_{10}(u)$ and $p_{11}(u)$, and yields the corresponding expression in Theorem 3.1.

Appendix D. Proof of Theorem 3.2

By the law of iterated expectation,

$$\begin{aligned}\mathbb{E}[P_A(t)] = \Pr\{X(t)=0\}\,&\mathbb{E}[P_A(t)\mid X(t)=0]\\ &+\Pr\{X(t)=1\}\,\mathbb{E}[P_A(t)\mid X(t)=1].\end{aligned} \tag{D.1}$$

Use the two–state transition probabilities to propagate the initial distribution, $p_0(0)=\Pr\{X(0)=0\}$: From A.5 and A.6

$$\Pr\{X(t)=0\} = \frac{\mu}{\lambda+\mu} + \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big)e^{-(\lambda+\mu)t}, \tag{D.2}$$

$$\Pr\{X(t)=1\} = \frac{\lambda}{\lambda+\mu} - \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big)e^{-(\lambda+\mu)t}. \tag{D.3}$$

Substitute these weights together with the conditional values from Theorem 3.1,

$$\mathbb{E}[P_A(t)\mid X(t)=0] = e^{nt}\Big[\frac{\bar\pi}{r-n} - \frac{\lambda}{\lambda+\mu}\cdot\frac{\Delta\pi}{r-n+\lambda+\mu}\Big], \tag{D.4}$$

$$\mathbb{E}[P_A(t) \mid X(t)=1] = e^{nt}\Big[\frac{\bar{\pi}}{r-n} + \frac{\mu}{\lambda+\mu}\cdot\frac{\Delta\pi}{r-n+\lambda+\mu}\Big]. \tag{D.5}$$

Thus

$$\mathbb{E}[P_A(t)] =$$
$$\Big(\frac{\mu}{\lambda+\mu} + \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big)e^{-(\lambda+\mu)t}\Big)\ e^{nt}\Big[\frac{\bar{\pi}}{r-n} - \frac{\lambda}{\lambda+\mu}\cdot\frac{\Delta\pi}{r-n+\lambda+\mu}\Big]$$
$$+\Big(\frac{\lambda}{\lambda+\mu} - \Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big)e^{-(\lambda+\mu)t}\Big)\ e^{nt}\Big[\frac{\bar{\pi}}{r-n} + \frac{\mu}{\lambda+\mu}\cdot\frac{\Delta\pi}{r-n+\lambda+\mu}\Big]. \tag{D.6}$$

Factor the common e^{nt} and expand the product:

$$\mathbb{E}[P_A(t)] = e^{nt}\frac{\bar{\pi}}{r-n}\Big(\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\Big)$$
$$+e^{nt}\frac{\Delta\pi}{r-n+\lambda+\mu}\Big[-\frac{\mu}{\lambda+\mu}\cdot\frac{\lambda}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\cdot\frac{\mu}{\lambda+\mu}\Big]$$
$$+e^{nt}e^{-(\lambda+\mu)t}\Big(p_0(0)-\frac{\mu}{\lambda+\mu}\Big)\Big[\frac{\bar{\pi}}{r-n}(1-1)-\frac{\Delta\pi}{r-n+\lambda+\mu}\Big(-\frac{\lambda}{\lambda+\mu}-\frac{\mu}{\lambda+\mu}\Big)\Big]. \tag{D.7}$$

Now simplify each line. First, $\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} = 1$, so the first term is

$$e^{nt}\frac{\bar{\pi}}{r-n}. \tag{D.8}$$

Second, the bracket in the second line cancels exactly (the two products are equal and opposite), so that whole line is zero. The remaining contribution comes from the, $e^{-(\lambda+\mu)t}$, term. Observe that,

$$-\frac{\lambda}{\lambda+\mu} - \frac{\mu}{\lambda+\mu} = -1, \tag{D.9}$$

so the third line reduces to,

$$e^{nt}e^{-(\lambda+\mu)t}\Big(p_0(0) - \frac{\mu}{\lambda+\mu}\Big)\frac{\Delta\pi}{r-n+\lambda+\mu}. \tag{D.10}$$

Collecting the non-zero pieces yields

$$\mathbb{E}[P_A(t)] = \frac{\bar{\pi}\, e^{nt}}{r-n} - \frac{\Delta\pi\, e^{(n-(\lambda+\mu))t}}{r-n+\lambda+\mu}\left[p_0(0) - \frac{\mu}{\lambda+\mu}\right], \tag{D.11}$$

which is the expression stated in Theorem 3.2.
Next, we derive the expression for the multiplier,

$$M(t) := \frac{\Delta P(t)}{\Delta\pi(t)}. \tag{D.12}$$

The numerator is computed by subtracting $\frac{\pi_0(t)}{r-n}$ from $\mathbb{E}[P_A(t)]$,

$$\mathbb{E}[P_A(t)] - \frac{\pi_0(t)}{r-n} = \frac{\lambda}{\lambda+\mu}.\frac{\Delta\pi(t)}{r-n} - \frac{\Delta\pi(t)\, e^{(n-(\lambda+\mu))t}}{r-n+\lambda+\mu}\left[p_0(0) - \frac{\mu}{\lambda+\mu}\right]. \tag{D.13}$$

The multiplier is obtained by dividing this quantity by $\Delta\pi(t)$. This gives,

$$M(t) = \frac{\lambda}{\lambda+\mu}\cdot\frac{1}{r-n} - \frac{e^{-(\lambda+\mu)t}}{r-n+\lambda+\mu}\left[p_0(0) - \frac{\mu}{\lambda+\mu}\right]. \tag{D.14}$$

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