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Cournot equilibrium at the limit

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Abstract

This paper studies a Cournot market with infinitely many firms facing constant but heterogeneous marginal costs, without assuming perfect competition. We determine a necessary and sufficient condition for the existence of equilibrium - the marginal costs converge to a limit r with summable deviations. We deduce from this condition that perfect competition is not automatic in such markets, but competitive behavior emerges asymptotically under certain conditions on the costs. We also consider a family of finite markets growing to the infinite limit market. We show that the equilibria of finite markets converge to that of the limit market if and only if the average marginal costs of the finite markets converge to r.

Keywords:
Cournot-Nash Equilibrium, Limit Market, Equilibrium Convergence

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1. Introduction

Classically, a market with infinitely many firms is considered perfectly competitive. The underlying rationale for this argument is that the output of each firm is negligible in comparison to the output of the entire economy, cf. for instance Hart (1979) [3]. This idea is widely used in various models both theoretical and empirical, supporting an infinite competitive market.

In this note, we consider this question from purely a mathematical point of view.

We consider a market $M(0)$ with countably infinite firms F_i (for $i \in \mathbb{N}$) facing heterogeneous but constant marginal costs c_i. We think of $M(0)$ as a Cournot game and derive a functional analytic condition characterizing the existence of equilibrium in $M(0)$. We show that a Cournot equilibrium in $M(0)$ exists if and only if the sequence $\{c_i\}$ of marginal costs converges to a limit r such that $\sum_i |r - c_i| < \infty$, or equivalently the sequence $\{r - c_i\} \in l^1$. Thus, the existence of equilibrium not only requires the marginal costs to concentrate near a value r, but to do so in a way that the aggregate deviations are summable. We work under the assumption that the aggregate output of the market is finite. The market $M(0)$ is a theoretical limit market that approximates large finite markets. In such markets, the market demand is finite, which leads to finite aggregate output as well, consistent with our assumption.

The equilibrium condition also endogenously determines the market clearing price as the limit cost r. So, the firms do act as price takers at equilibrium. However, those with significantly lower costs $c_i \ll r$ still make profits and have a significant market share in terms of output and revenue. In fact, in contrast to the classical arguments, we show that it is possible for the firms at the head of the sequence to have a market share comparable to the aggregate market output. In addition, at equilibrium, Price $\neq$ Marginal cost for the firms. Thereby, this market $M(0)$ is not perfectly competitive in the classical sense. Nevertheless, the cost heterogeneity among the firms vanishes in the tail, and the market asymptotically approaches perfect competition. This can be considered an intermediate market between the perfectly competitive market and an oligopoly.

Next, we consider a family of growing but independent finite markets $M(\alpha)$ parametrized by $\alpha \in (0, 1]$ each with $\lfloor \frac{1}{\alpha} \rfloor$ firms. We identify the conditions for the equilibria of the family $M(\alpha)$ to converge to that of $M(0)$

in terms of aggregate quantity. This model is similar to that of Novshek [5] (1985), except that we do not a priori assume that the limit market $M(0)$ is perfectly competitive. We show that a necessary and sufficient condition for the equilibria convergence is that the generalized sequence of average marginal costs of $M(\alpha)$ converges to r, the limit marginal cost of $M(0)$. This result regarding equilibrium convergence justifies the use of the theoretical limit market $M(0)$ to represent large finite markets.

The results of Munter [4] (2017) show that cost asymmetries between firms are an entry barrier to the market and a source of persistent market power. This complements our conclusion that in order for the market $M(0)$ to support infinitely many firms at equilibrium, the cost heterogeneity must eventually become negligible. The market conditions that lead to perfect competition have also been investigated in some prior works although in different settings and models. For instance, Bresnahan and Reiss [1] (1991) show empirically that market homogeneity emerges when the number of firms increases even slightly. Another instance is the work of Peretto [6] (1999), who in a dynamic growth model shows that growth in terms of scale and population leads to stabilization of market behavior. Our approach, on the other hand, uses only mathematical analysis to derive a simple condition that ensures the emergence of equilibrium in an infinite market by relating it with the distributional behavior of costs. This condition refines the classical premise that perfect competition is an automatic implication of the presence of a large number of firms, and shows that it is actually the vanishing of heterogeneity which leads to competitive behavior. Even in that case, oligopolistic features in the market persist.

Thus this paper gives a mathematical foundation to the classical argument in Industrial Organisation that a market with infinitely many firms is perfectly competitve, by showing precisely how the competitive behavior emerges in terms of a mathematical condition, and indeed how it is not automatic.

2. The Model

We consider a family of markets $M(\alpha)$ where $\alpha \in [0, 1]$, all producing the same homogeneous product. The features of these markets are as follows.

2.1. Number of firms

The market $M(\alpha)$ for $\alpha > 0$ has $\left\lfloor \dfrac{1}{\alpha} \right\rfloor$ firms. Note that $\left\lfloor \dfrac{1}{\alpha} \right\rfloor$ represents the greatest integer $\leq \dfrac{1}{\alpha}$. So each market $M(\alpha)$ has finitely many firms and the number increases as $\alpha \to 0$.

2.2. Output, Demand and Cost

For $\alpha > 0$, the market $M(\alpha)$ has firms $F_{\alpha,i}$ ($i = 1, 2, \cdots, \lfloor \frac{1}{\alpha} \rfloor$) with marginal costs $c_{\alpha,i}$ ($i = 1, 2, \cdots, \lfloor \frac{1}{\alpha} \rfloor$). These firms use pure strategy Nash equilibrium to determine their outputs $q_{\alpha,i}$. The market has a linear inverse demand $P_\alpha = a - b \displaystyle\sum_{i=1}^{\lfloor \frac{1}{\alpha} \rfloor} q_{\alpha,i}$ with $a, b > 0$.

2.3. Limit Market

The market $M(0)$ has countably many firms F_i where $i \in \mathbb{N}$, with marginal costs c_i. We do not assume that this is a perfectly competitive market. Firm F_i chooses an output q_i. The inverse demand in this market is $P = a - b \displaystyle\sum_{i=1}^{\infty} q_i$ which presupposes the finiteness of the aggregate output $\displaystyle\sum_{i=1}^{\infty} q_i$

2.4.

The parameters a, b are held constant across all markets $M(\alpha)$ reflecting a stable demand structure as the market grows.

2.5.

The markets $M(\alpha)$ are independent; firm identities may vary with α signifying free entry and exit.

2.6.

We treat each market $M(\alpha)$ $(\alpha \in [0, 1])$ as a Cournot game where the firms compete to set profit maximizing quantitites at equilibrium.

2.7.

For $\alpha > 0$, denote $\left\lfloor \frac{1}{\alpha} \right\rfloor =: N$. Then the following results are well-known at equilibrium for the finite market $M(\alpha)$, cf. [2].

1. The aggregate output is $Q_\alpha = \displaystyle\sum_{i=1}^{N} q_{\alpha,i} = \frac{Na - C_\alpha}{(N+1)b}$. Here $C_\alpha = \displaystyle\sum_{i=1}^{N} c_{\alpha,i}$ represents the total marginal cost in $M(\alpha)$.

2. Output of firm $F_{\alpha,i}$ is $q_{\alpha,i}$ and $bq_{\alpha,i} = \dfrac{a + C_\alpha}{N+1} - c_{\alpha,i}$.

3. The equilibrium price $P_\alpha = a - b\displaystyle\sum_{i=1}^{N} q_{\alpha,i} = \frac{a + C_\alpha}{N+1}$.

3. Existence and the structure of equilibrium in the limit market $M(0)$

Classically a market with countably infinite firms is viewed as perfectly competitive, since each firm's market share is considered infinitesimal. We do not a priori start with this assumption but instead derive conditions under which this limit market has a Cournot equilibrium.

Recall from § 2.3 that the market $M(0)$ has firms F_i ($i \in \mathbb{N}$) with marginal costs c_i. We consider the market to be a Cournot game for outputs q_i. We make the following assumption.

(A1) The sequence $(q_n) \in l^1$. Note that l^1 is the set of all sequences which are absolutely convergent i.e. $l^1 = \{(x_n) : \sum_{n=1}^{\infty} |x_i| < \infty\}$. Our assumption is that the aggregate output in the market $M(0)$ is finite. This is reasonable since the market output has to just clear the market demand, which we expect is finite.

The inverse demand function determined by the total output is $P = a - b\displaystyle\sum_{n=1}^{\infty} q_n$. The profit of firm F_i is $\pi_i = Pq_i - c_iq_i = (a - b\displaystyle\sum_{n=1}^{\infty} q_n - c_i)q_i$. Note also that π_i is a differentiable function of q_i because of the assumption (A1).

Theorem 1. *The limit market $M(0)$ has a Cournot equilibrium if and only if there exists $r \in \mathbb{R}$ satisfying $(r - c_i) \in l^1$, i.e. $\sum_{i=1}^{\infty} |r - c_i| < \infty$. Consequently, $\lim_{n\to\infty} c_n = r$. Hence $M(0)$ has an equilibrium if and only if the marginal costs converge to a limit cost r with summable deviations.*

Proof. The equilibrium is an output sequence $(q_n) \in l^1$ which simultaneously satisfies the first order conditions given by equations

$$\frac{\partial \pi_i}{\partial q_i}((q_n)) = 0 \text{ for } i = 1, 2, 3, \cdots$$

Solving these gives

$$q_i = \frac{a - b\sum_{n=1}^{\infty} q_n - c_i}{b}\,.$$

Now the equilibrium (q_n) exists $\iff$ the sequence $\left(\left(\frac{a - b\sum_{n=1}^{\infty} q_n - c_i}{b}\right)\right) \in l^1$ by assumption (A1). Denote $\sum_{n=1}^{\infty} q_n =: Q$ at equilibrium. Then, equilibrium exists

$$\iff \sum_{i=1}^{\infty} a - bQ - c_i < \infty\,.$$

In particular, equilibrium exists $\iff$ there is a $r \in \mathbb{R}$ such that $\sum_{i=1}^{\infty}(r - c_i) < \infty$, i.e. $(r - c_i) \in l^1$ implying that $\lim_{i\to\infty} c_i = r$. □

3.1. The constant r

The term $(r - c_i)$ represents the deviation of the marginal cost of F_i from the limit marginal cost. Theorem 1 implies that while the costs may be heterogeneous, the equilibrium exists only if these differences become negligible eventually, and further they sum to a finite quantity.

Theorem 2. *The endogenously determined price P at the equilibrium is the limit marginal cost r.*

Proof. From Theorem 1, the equilibrium exists $\iff$ $a - bQ - c_i \in l^1$, i.e. $\lim_{i\to\infty} c_i = a - bQ$. By uniqueness of limits, $a - bQ = r$. Thereby, at equilibrium, the aggregate quantity is

$$Q = \frac{a - r}{b}\,,$$

and the price is $P = a - b\left(\frac{a - r}{b}\right) = r\,.$ □

3.2. Features of equilibrium structure on $M(0)$

1. The output of firm F_i is $q_i = \dfrac{a - b\sum_{n=1}^{\infty} q_n - c_i}{b} = \left(\dfrac{r - c_i}{b}\right)$.

2. Since $r = \lim_{n\to\infty} c_n$, the marginal costs c_n become arbitrarily close to r for large n. Hence, for large n, the output q_n and the profit $\pi_n = (P - c_n)q_n = (r - c_n)q_n$ of F_n are negligible. These could represent firms with small scale operations with a modest aim to break even or do just better.

3. However, this does not deter finitely many firms at the beginning of the sequence from having marginal costs substantially lower than r i.e. $c_n \ll r$. This allows for the possibility of finitely many firms to have high profits, and high output comparable in relation to the aggregate output.

4. To quantify how much the market $M(0)$ at this equilibrium behaves like perfect competition, we look at the distribution of firms in the error sets
$$E(\epsilon) = \{i \in \mathbb{N} \,:\, |r - c_i| < \epsilon\}\,.$$
The set $E(\epsilon)$ contains firms in the tail of $M(0)$ which are ϵ close to the limit cost r. In light of the previous theorem, the sets $E(\epsilon)$ contains all but finitely many firms which are approaching perfect competition. But the finitely many firms outside these error sets are the ones that wield some market power.

In the infinite market $M(0)$, all firms are price takers. This market asymptotically approaches perfect competition – marginal costs converge, so the cost heterogeneity and the market power eventually vanish. However, it is possible, for finitely many firms with superior technology or cost advantages to dominate the market in production and profits. This gives rise to an intermediate market structure - competitive in the limit, but with residual oligopolistic behavior in the head.

4. Equilibrium convergence

We now consider the convergence of equilibrium of the markets $M(\alpha)$ to the that of $M(0)$ in terms aggregate output or demand convergence.

The aggregate outputs $\{Q_\alpha\}$ for $\alpha \in (0,1]$ form a generalized sequence or a net indexed by the directed set $(0,1]$. Similarly the demand functions $\{P_\alpha\}$ of the markets $M(\alpha)$ for $\alpha \in (0,1]$ form a generalized sequence. Convergence theory for generalized sequences is very similar to that of sequences.

Note that the generalized sequence of aggregate outputs $\{Q_\alpha\}$ of $M(\alpha)$ converges to the aggregate output Q of $M(0)$ if and only if the demands $\{P_\alpha\} = \{a - bQ_\alpha\}$ of $M(\alpha)$ converges to $P = a - bQ$, the demand of $M(0)$.

Now we deal with the convergence of equilibria of $M(\alpha)$ to that of $M(0)$ in terms of aggregate outputs (equivalently aggregate demands) and relate it to a condition on the marginal costs.

Theorem 3. *The generalized sequence $\{Q_\alpha\}$ of aggregate outputs of $M(\alpha)$ converges to the aggregate output $Q = \dfrac{a-r}{b}$ of $M(0)$ if and only if the generalized sequence of average marginal costs of $M(\alpha)$ converges to r as $\alpha \to 0$. Precisely, the equilibrium convergence happens if and only if*

$$\lim_{\alpha \to 0} \frac{C_\alpha}{\left\lfloor \frac{1}{\alpha} \right\rfloor} = r,$$

where $C_\alpha = \sum_{i=1}^{\left\lfloor \frac{1}{\alpha} \right\rfloor} c_{\alpha,i}$ is the total marginal cost in $M(\alpha)$.

Proof. Recall that the equilibrium output $Q_\alpha = \dfrac{\left\lfloor \frac{1}{\alpha} \right\rfloor a - C_\alpha}{(\left\lfloor \frac{1}{\alpha} \right\rfloor + 1)b}$. Since $\lim_{\alpha \to 0} \frac{\left\lfloor \frac{1}{\alpha} \right\rfloor a}{(\left\lfloor \frac{1}{\alpha} \right\rfloor + 1)b} = \frac{a}{b}$, the generalized sequence $\{Q_\alpha\} \to Q \iff \lim_{\alpha \to 0} \frac{C_\alpha}{\left\lfloor \frac{1}{\alpha} \right\rfloor + 1} = r$.

Also, $\lim_{\alpha \to 0} \frac{C_\alpha}{\left\lfloor \frac{1}{\alpha} \right\rfloor + 1} = \lim_{\alpha \to 0} \frac{C_\alpha}{\left\lfloor \frac{1}{\alpha} \right\rfloor} = r$ since $\lim_{\alpha \to 0} \frac{\left\lfloor \frac{1}{\alpha} \right\rfloor}{\left\lfloor \frac{1}{\alpha} \right\rfloor + 1} = 1$. Hence $\{Q_\alpha\} \to Q$ if and only if the average costs $M(\alpha)$ converges to the limit marginal cost of $M(0)$. $\square$

We now note the following inter-related points.

1. We infer from Theorem 3 that the average marginal costs of markets of large size, i.e. the average marginal cost of $M(\alpha)$ for small α is approximately r.

2. On the other hand, note also that in the market $M(0)$, $\lim_{i \to \infty} \dfrac{\sum_{n=1}^{i} c_n}{i} = r$. Indeed, Theorem 1 shows that the limit market $M(0)$ has a Cournot equilibrium if and only if the cost sequence $\{c_i\}$ of the firms converge to

a limit r with $\sum_{i=1}^{\infty} |r - c_i| < \infty$. Since $\lim_{i\to\infty} c_i = r$, the cost sequence $\{c_i\}$ is also Cesaro summable to r, which gives $\lim_{i\to\infty} \frac{\sum_{n=1}^{i} c_n}{i} = r$ that is mentioned above. This means, if we truncate the tail of $M(0)$ to leave behind a large finite market, the average marginal cost of this large finite market is approximately r as well.

These two together clearly imply that large finite markets $M(\alpha)$ are effectively approximated by the limit market $M(0)$ in terms of average marginal costs at equilirbium.

5. Conclusion

The contributions of this paper are twofold. Firstly, the paper gives a concrete instance where a market with infinitely many firms is not perfectly competitive. In particular, we recognise that if the marginal costs $\{c_i\}$ of the infintely many firms are such that $\{r - c_i\} \in l^1$ for some number r, then the market $M(0)$ is in equilibrium. But, as is common with convergence conditions, this only imposes restrictions on the tail of the sequence $\{c_i\}$ and allows for finitely many firms to have costs c_i significantly lower in comparison to r. In such a case, the output of these firms are the significant contributors to the aggregate output of the market $M(0)$. The contributions of the tail firms to the aggregate output is negligible. This allows for finitely many firms to have significant market share. So these finitely many firms give the market an oligopolistic structure. We also observe that when cost differences vanish at the tail, competitive behavior emerges. So this model also illustrates that large markets become competitive asymptotically not just because of infinitesimal firm size but because heterogeneity vanishes with scale. This setup also illustrates how large markets can simultaneously display both competitive and oligopolistic features.

Secondly, § 4 shows that very large markets indeed behave similarly to the limit market in terms of aggregate output and average marginal costs when there is equilibrium convergence. Thus, the limit market $M(0)$ offers an accurate approximation of large economies, justifying its theoretical relevance.

We remark that these results are easily extendable to non-linear demand functions like $P = a - b \sum_i q_i^n$, and perhaps to more general demand functions as well.

In conclusion, the paper mathematically clarifies the classical understanding of the competitive nature of an infinite market. It determines precisely how this competitive behavior emerges as a result of diminishing heterogeneity of costs, thus revealing the theoretical subtlety underlying the competitive limit.

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